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A journey to the reappraisal of the term 'student-centred'

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Abstract

The article is based on a research in which the primary aim of the study had been to find empirical justification for seven crucial challenges that could be considered simultaneously within instrumental orchestration [1]. The results of the data synthesis suggested that a quasi-systematic framework to promote links between conceptual and procedural knowledge may be crucial when planning the problems of students' own investigation processes, whereby an innate way to utilize technology is to proceed in a more or less non-systematic way at the level of instrumentalisation (i.e. knowledge construction means bi-directional actions between the person and the tool). As students' freedom to choose learning objectives and working methods appeared in a most natural way in collaboration between students or student teams, the research process offered a journey to take a critical position on the term 'student-centred', being characterized in a more or less "loose and grey" way in the literature. The synthesis reveals that Neuman's [2] triple-step contexts for defying "student-centred learning" should extended by the paradigm between student teams. Thus, rather than trying to be written as a rigid research report, this article describes this journey in a way that hopefully portrays the complexity of the field and emphasizes the challenge to consider several components simultaneously. Implications for teacher education and school culture are discussed briefly.

1. Introduction

1.1. Researchers' efforts to characterize the term SCL

The term Student-centred Learning (SCL) has turned out to be a complicated and messy concept (see [3]). Beyond the efforts to define it as the opposite of teacher-centred learning ([4], [5], [6], [7], [8]), it seems to appear repeatedly as "loose" attributes as 'active learning', 'choice in learning', 'self-guidance', 'autonomy', and 'collaboration' (cf. [9], [10], [11], [12], [13]). According to [14], cooperative work is accomplished by dividing the labour among participants as an activity in which each person is responsible for a portion of the problem solving, whereas collaboration involves the mutual engagement of participants in a coordinated effort to solve the problem together. Self-guidance means that learners participate in guiding and planning their own learning, like defining homework [15]. These elements improve the learner's intrinsic motivation toward learning and can be seen to create better learning results. Many researchers have reported that the feeling of self-guidance improves learners' meta-cognitive skills and ability to evaluate their own work ([16], [17], [18]). In contrast, the teacher-centred, well-controlled approach decreases learners' intrinsic motivation towards and involvement in working [19]. The term "self-guidance" refers to similar processes as the concept of self-regulated learning (cf. [20], [11]) promoting an opportunity to consider the process phases (preparatory, performance and appraisal) with different emphasis [21].

It seems natural to require that student-centred approaches should allow the students at least a certain level of cooperation and even collaboration and to make their own decisions regarding their own learning (cf. [22]). Zain, Rasidi and Abidin [23] suggest that this shifts the students from being less passive receivers of knowledge to being more responsive, and able to relate to their experiences. Students' learning skills are demonstrated through their heightened interaction and cooperation (in and outside the classroom), better planning of the lesson and students' learning, with some elements of analytic skills being portrayed. Many studies also highlight aspects of curriculum design and assessment: there is a gap between what students expected in their learning and what teachers have taught them. Saragih & Napitupulu [24] report on the impact of the positive effect on students' higher order skills and communication, whilst Noyes [25] and Calder [9] suggest that the nature of scaffolding might differ from that experienced in traditional classroom situations. This is in accord with widely used demands to emphasize student preparation for college and careers with relevant and flexible knowledge, as learning can happen at any time and anywhere (see [26]). Overall, the following quotation from Swan [27] might serve as a summary of the characterizations found for SCL, especially in mathematics:

> *"the teacher takes students' needs into account when deciding what to teach, treats students as individuals rather than a homogeneous body, is selective and flexible about what is covered and allows students to make decisions, compare different approaches and create their own methods. Instead when using teacher-centred practices, the teacher directs the work, pre-digests and organizes the material, gives clearly prescribed instructions, teaches everyone at once in a predetermined manner and emphasizes practice for fluency over discussion for meaning*."

By extending Neuman's three-step paradigm (*on*, *in*, and *with*) [2], we will extract ourselves from this position in the conclusion of our study, not the least because we doubt whether mathematical knowledge can be constructed by an individual in an optimal way by solving more or less closed tasks that someone else has posed in a well-defined and well-organized mathematical content. Even the very basic feature of constructivism requires the teacher to utilize dialectic problem posing (see [28]), which acts as a form of vaccination against negative emotions.

By making synthesis from the results of the previous research [1] we will pose in this article a research question: (Q) What kind of demands could be set for the student centred learning environments (SCLE). To do this end we need to look for closer and widely the theoretical aspects concerning learning, learning environments, and mathematics as a subject to be study, and its specialties concerning knowledge and learning solutions (Section 2). Furthermore, we will promote our findings from the ClassPad project [1] that are reflecting the presented theoretical network (Section 3). Finally, we are ready for presents our synthesis concerning SCLE on Section 4.

2. Background

2. 1. Challenges caused by the complexity of the field

To begin to reappraise the term 'student-centred learning', the first requirement was to take a critical look at the use of the term "learning", which in itself is always essentially "student-centred". It seems strange even to suggest that someone is learning on behalf of another person, and for example, it might be more appropriate to speak about "student-centred teaching" (cf. [25]), "student-centred instruction" (cf. [47]), or "student-centred learning environments" (SCLE).

To replace these loose characterizations, we need grounding on sustainable and viable frameworks for instructional praxis. Because of the complexity of the field (cf. [48]) we had to withdraw from the tradition of focusing on a single factor or on a few partial factors of mathematical instruction at a time. We have adopted the views of [49] and [50]) that the following seven challenges of instrumental orchestration should be considered more or less simultaneously:

1. Promoting collaborative social constructions;
2. Linking of conceptual and procedural knowledge;
3. Solving the dilemma between a systematic approach and minimalist instruction;
4. Relating instructional design and assessment to instrumental genesis;
5. Promoting learning by design;
6. Revitalizing sustainable heuristics in human history; and
7. Applying business principles to overcome the bad reputation of mathematics.

The matrix for SCLE to be represented in the Conclusions section (Table 4), for example, should be understood as a general framework only. The analysis of each of the cells requires that the seven challenges of instrumental orchestration are more or less related to each other. However, as it is impossible for this to be reported within the scope of this article, we represent the so-called ClassPad project from the viewpoint of those challenges in the sections that follow. Thus, instead of being a formally reported empirical study, this article may appear rather as a journey to Grounded Theory, finally leading to a reappraisal of SCL and SCLE.

2.2. Promoting collaborative social constructions

When considering the learning of mathematics within a constructivist paradigm, instead of speaking about 'learning environments', it might be more relevant to adopt the term *investigation space,* as used by Haapasalo &Samuels [51] whereby the learning is considered to be an investigation process – whether individual or collaborative - including both cognitive and psychological aspects. The term 'space' accents that this process that is nowadays independent of time, place and formal modes and emphasizes students' own freedom and control. To make an appropriate *socio-constructivist* grounding, we have adopted the well-known *pragmatic theory of truth* emphasized by the philosopher Charles Peirce. This theory is as valid for scientists as for learners because when an investigation space has been designed (an open dialectic problem is given; see [28]), the teams collaborate in causal interaction with this problem. After testing the viability of radical ideas within the teams and between the teams, only those ideas that are viable for the whole social group consisting of those teams (see Figure 1) still remain. The objectivity of knowledge is related to what the teacher and students see as necessary "to be able to cope" in the sense of von Glasersfeld [52]. We will apply this paradigm throughout our study and when reappraising SCLE in the conclusion.

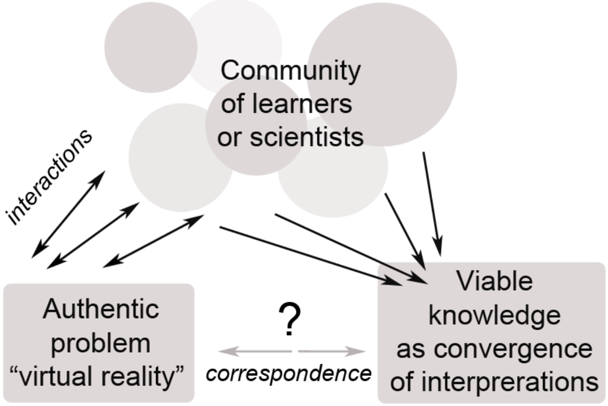

Figure 1. Viable knowledge as a result of radical social constructions (cf. [49]).

2.3 Sustainable heuristic activities may be supported in new way

To consider how mathematical knowledge and mathematical thinking enters the human mind and life and enables coping in the sense of Figure 2, it is appropriate to recognize which heuristic activities have been sustainable throughout human history. During his long-term study of the history of mathematics, Zimmermann ([53], [54]) identified eight main activities which have often led to mathematical innovations over different times and cultures for more than 5,000 years. We will henceforth refer to the Z-activities, represented in Figure 1.

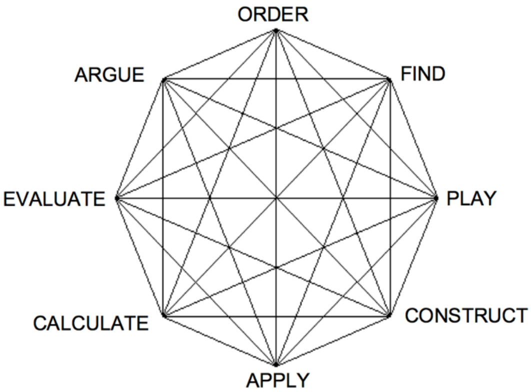

Figure 2. Activities and thinking tools which proved to be successful in mathematics making [54].

The meta-study by Haapasalo and Hvorecky [55] gives a comprehensive evaluation of Z-activities based on well-established criteria for the quality of research in mathematics education (see [56]). They emphasize the significance, rigor, and both theoretical and pragmatic relevance of Z-activities, which can be linked to Bishop's [57] comprehensive analysis of educational consequences from sociocultural perspective, as to his "concept-based components" counting, locating, measuring, designing, playing, and explaining. On the other hand, measuring, for example, comprises almost all the Z-activities and their linkages.

When reflecting upon Z-activities from an educational point of view, it has been recognized in many empirical studies (cf. e.g. [58]) that they are just as important for today's mathematics and science instruction, especially if the creative activities of pupils are stressed. The interconnections between these activities, represented in Figure 2, correspond to the general goal (of learning) to achieve a high degree of flexibility in thinking, and to foster connected, divergent thinking in addition to mastering routine activities. They are also in accord with the well-known attributes for expert-like working and thinking (see e.g. [59], [60]).

To find out how Z-activities are supported, we developed a 5-step Likert scale instrument to measure the following three profiles: (1) *Self-confidence:* How strongly the student thinks he or she is performing each of the activities, (2) *Maths profile*: How strongly the student thinks mathematics teaching supports each of the activities, and (3) *ICT profile:* How strongly the student thinks the usage of ICT supports each of the activities - where ever and how ever he or she uses it (see Figure 2). Later, the instrument was developed by representing each of the main activites as three sub-activities (see [61]). When measuring the three profles among prospective mathematics teachers and prospective elementary teachers, Haapasalo and Eskelinen [61] found that the only note-worthy support gained from mathematics teaching at school or university, seems to come for calculating. Surprisingly the support the students thought to have gained from their own usage of information and communication technology (ICT), when ever and how ever they used it, was even more modest. However, in the next section we provide an example of how students could utilize techology in their free time with well-tailored investigation tasks for the learning of mathematics in a way that extended all three aforementioned profiles.

2.4 Shifting from instrumentation to instrumentalisation

We have used the term *instrumental genesis* in a wide sense to mean the development of ICT together with its usage for acquisition of conceptual and procedural knowledge. It comprises two parallel components: *instrumentation* and instrumentalisation ([62], [63]). The former refers to a person's ability to use a tool. It is directed towards an artefact and describes the process by which it becomes useful to the learner to accomplish specific purposes (that is, an instrument). The latter refers to the way a person uses a tool to shape the actions and the character of the knowledge constructed with the tool. It is directed towards the learner and describes the process by which the opportunities and constraints of the artefact shape their conceptual understanding and procedural ability. Instrumentation and instrumentalisation often happen naturally in students' free time as they tailor their smartphone and tablet apps in creative ways for their own purposes. This suggests that mathematical instruction should shift its focus from well-prepared classroom lessons to *instrumental orchestration*. We use this term, introduced by Trouche [64], to mean intentional and quasi-systematic organization of available ICT tools within an appropriate paradigm of teaching and learning to promote students' instrumental genesis. By using the term "quasi", we emphasize two aspects from the teacher's side: firstly, the need to plan the learning environments systemically, based on viable and sustainable theories of teaching and learning; and secondly, the need to accept the principle of minimalist instruction (see [65]) because the learning very often proceeds more or less spontaneously. The integrated environment of a *computer algebra system* (CAS) and a *dynamic geometry* (DGS), for example, allows casual playing between mathematical representations offering a powerful tool for problem solving and promotion of links between procedural and conceptual knowledge.

Eronen and Haapasalo [66] report how students could utilize such technology in their free time with well-tailored investigation tasks in a way that extended their all three profiles mentioned earlier, even during a short period of working time. At the beginning of the so-called *ClassPad Project*, the unfamiliar calculator (https://edu.casio.com/products/cg/cp330plus/) was demonstrated briefly to a class of Year 8 students (N=15) to give them the opportunity to play with it voluntarily during their summer holiday with concepts of Year 9 mathematics (such as a linear functions). Their only requirement was to write a portfolio of reflective notes if they worked with the tool. The authentic sample shown in Figure 3 shows a sample, having been made at 1:42 a.m. during a working period of 75 minutes, continuing the next day when the student explained how the parameters affect the position and location of the line:

- The equation is now y = 2x **- 3.**
- When changing the equation to y = 2x **– 5,** the line moves forwards in the positive direction of the x-axis.
- By changing the equation to y = 2x – **1,** the line has gone in the same direction as before, but the distance is smaller.
- I notice I solved the task in a strange way. I'm glad that I understood these cases and now my studying is going pretty well.

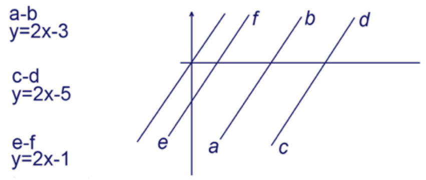

Figure 3. An example of instrumentalisation with ClassPad (cf. [50])

This example contrasts with the common purely metacognitive abilities of students and teachers [67]. By manipulating the conceptual interpretation spontaneously, the student explained how the parameters affect the position and location of the line (procedural interpretation). Through instrumentalisation, she made her own interpretation against the standard view: the line moves along the horizontal axis.

2.5 Promoting links between conceptual and procedural knowledge

The characterization of these two knowledge types has been identified as a neglected area in mathematics education research — even though it is a key question in any pedagogy to ask whether the learner must understand before being able to do, or vice versa. This means solving the conflict between conceptual and procedural knowledge ([31], [32], [33], [34]), characterized according to Haapasalo and Kadijevich [68] as follows:

- *Procedural knowledge denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.*

- *Conceptual knowledge denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.*

Recalling their discussion about the importance of procedural knowledge in human constructions of meaning, investigation spaces should allow learners to start from their *spontaneous procedural knowledge*. However, procedural knowledge alone cannot predominate if we consider that the main goals of education are to promote skilful navigation in knowledge networks, and the ability to apply knowledge in new situations, requiring linkage between Z-activities. Research by Lauritzen [69] with economics undergraduate students (n = 476) reveals two crucial factors in acquiring and applying knowledge. First, procedural knowledge is necessary but not sufficient for conceptual knowledge; and second, to be able to apply what they know, students need conceptual knowledge. Combining these demands, we can conclude that the so-called *developmental approach*, based on a genetic view emphasizing procedural knowledge, needs to be combined with an *educational approach*, based on dynamic interaction and emphasizing conceptual knowledge (see [68]). Figure 4 represents a quasi-systematic model for a sophisticated interplay of the two approaches.

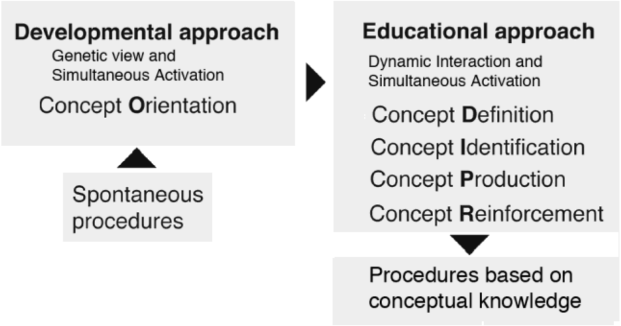

Figure 4. Interplay between developmental and educational approaches (cf. [49]).

When planning a constructivist approach to the mathematical concepts under consideration, the focus is on the left-hand side when the students try to interpret a tailored problem situation based on more or less spontaneous procedural knowledge. On the other hand, when offering students opportunities to construct links between representation forms of a specific concept, the focus is on the right-hand box, in which the stages of mathematical concept building are illustrated. In learning situations, however, students must have freedom to choose the problems that they want to learn how to solve, accompanied by continuous self-evaluation instead of relying on the expressed guidance of teachers. The next Section includes an example to solve this dilemma.

2.6 Combining systematic planning and minimalism

Recalling Figure 2, it is the right-hand half of the octagon that emphasizes creative human activities, which often run optimally without any external instruction or requirement. Students frequently neglect teacher tutoring, or they feel they do not have time to learn how to use technical tools; similarly, teachers feel they do not have time to teach how these tools should be used. The term *minimalist instruction* (MI), introduced by Carroll [65] is crucial not only for teachers, but also for those who write manuals and help menus for software. Carroll observed that learners often avoid careful planning, resist detailed systems of instructional steps, tend to be subject to learning interference from similar tasks, and have difficulty recognizing, diagnosing, and recovering from their errors. The assumptions, characteristics, and methods of minimalism implemented here are opposite to those used by Gagné [70]. With a view to fostering problem-solving abilities, we have picked up some alternative characteristics of MI (cf. [71], [72]). These features of *minimalism* encompass several varieties of the constructivist view, and include certain assumptions about effective instruction (cf. [73], [74]): 1) specific content and outcomes cannot be pre-specified, although a core knowledge domain may be specified; 2) learning is modelled and coached for students with unscripted teacher responses; (3) learning goals are determined from authentic tasks stressing doing and exploring; (4) errors are not avoided but are used for instruction; (5) learners construct multiple perspectives or solutions through discussion and collaboration; (6) learning focuses on the process of knowledge construction and development of reflexive awareness of that process; (7) criteria for success are the transfer of learning and a change in students' action potential, and (8) the assessment is ongoing and based on learner needs [67].

The genesis of heuristic processes, and the ability of students to develop intuition and mathematical ideas within a constructivist or minimalist approach, is unlikely to be attainable without thorough planning of learning environments by the teacher. To this end, empirically tested and more or less systematic pedagogical models can be helpful. Kadijevich and Haapasalo [75] found that links between conceptual and procedural knowledge may be established by means of sophisticated conceptually-oriented technology-based environments. The example in Section 2.4 reinforces not only this but suggests that students are able to use the tool on the level of instrumentalisation.

3. Empirical evidence from ClassPad 2 project

3.1. Background

The previously mentioned ClassPad project (see details [1]) was designed to investigate the kinds of opportunities the CAS technology might promote for seven challenges of instrumental orchestration within technology-based learning environments (Section 1.1). Therefore, SCL came under scrutiny more or less incidentally.

In the ClassPad2 project, we orchestrated the learning of the core areas of 9th grade mathematics (i.e. linear functions and basic concepts of statistics) using the ClassPad calculator as the learning tool, without any textbooks or traditional homework (to be referred to later as instrumental orchestration). The learning tasks were designed within the quasi-systematic framework in Figure 4. The learning of linear functions consisted of nine 45-minute lessons. During the first lesson, the students formed teams and learned to use ClassPad. The focus was on changing representations between algebraic and geometric calculator windows by utilizing the principle of the simultaneous activation of conceptual and procedural knowledge (cf. portfolio sample in Figure 3). The learning tasks were designed within the quasi-systematic framework of Figure 4, whereby the links between verbal (V), symbolic (S), and graphic (G) representations are constructed at first on the level of identification, and then on the level production (P). However, the students had freedom to "jump the gun" by choosing from this so-called buffet any problem that they wanted [66]. To find empirical support for the characterization of SCLE, students' working processes during ClassPad 2 were looked from five points of view:

(Q1) Can a quasi-systematic framework to link conceptual and procedural knowledge be used within a minimalist approach to instruction?
(Q2) Which kinds of cognitive development can be found among students?
(Q3) What influences did SCLE have on the students' mathematical identity?
(Q4) How did the students communicate in SCLE, and
(Q5) How did the students experience SCLE?

Data for Q1 were gathered from students' lesson diaries (n=23), and Data for Q2 consist of linear equations pre-test, test, and post-test patterns [66]. These tests were designed according to the MODEM framework [78]. Data for Q3 were gathered from the whole group via a web-based questionnaire before and after the ClassPad project, and by interviewing students to gain more information beyond the shifts of students' Identity- and Maths-profiles during the project. Data for Q4 were gathered from eight student discussions during the ClassPad 2 project lessons [76].

Discussions were audio recorded and data were transcribed. Data for Q5 consists of students (n=23) wrote essays on their experiences. Data for Q1, Q3 and Q4 were analysed by using content analysis, Data for Q2 were analysed by quantitative methods and Data for Q5 were analysed by using the Grounded Theory method (see [1], [66], [76], [77]).

3.2 Reported results

Concerning the first research question (Q1), the results reveal that even though students proceeded more or less chaotically, it was found that learning to link conceptual and procedural knowledge can be planned within a quasi-systematic framework ([66], [1]). To “go for lines”, one student team, for example, initially selected a quite complicated problem on optimizing mobile phone costs, which was planned to be a reinforcement task. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set. This happened to consist of Identification Tasks – the lowest level of understanding the links between representations (see Figure 4). This example shows that a sophisticated interplay between a systematic and minimalist approach can be achieved even by simple pedagogical solutions. Note this important feature of Minimalism: The teacher did not want to regulate students’ work by recommending that they try an easier sub-problem, for example. Instead, it was students’ internal motivation that regulated their task choice.

The self-guided selection of problems from the buffet really varied among the students. Some students mentioned that they just randomly picked problems from the buffet, and some students carefully selected what kinds of problems they were able to solve. The problems should include a wide range of choices in terms of level of difficulty. Tasks that are too simple kill the joy of discovery for learners, and tasks that are too difficult are overly challenging. It seems that interplay between a systematic and minimalist approach can be achieved even by simple pedagogical solutions [76]. Eronen [1] (see p. 38 Figure 9) represents example of the path of a student team which did not utilize the MODEM framework in any optimal way. At first it went directly to the Production tasks and selected those tasks more or less randomly (as was the case within the other task types after that). The students evidently liked the amazing drag-and-drop function, which automatically performed the action from Symbolic to Graphic Form and vice versa.

Which kinds of cognitive development can be found among students?

To follow up on the students’ learning about linear functions (Q2), the students were tested three times by using MODEM 1 test patterns: before the project (pre-test), at the end of the project (test), and five months after the project (post-test) [66]. The Cronbach’s alpha, calculated from each test item showed a good reliability level in all three tests (see Table 1). After the working period, the students’ total scores in the test were significantly higher ($p < 0.001$, sign test) than in pre-test. Moreover, the students’ scores in the post-test showed that the student’s mastery of the concepts they had learned during the ClassPad project was still at a high level, as there was no significant difference ($p<0.383$) in the scores in the test and pre-test. The test results indicated that the students indeed learned the linear function concept during the ClassPad project [76].

Table 1. Students’ (n=23) performance during the ClassPad project [76].

	Pre-test	Test	Post-test	a) Average total test scores out of 60 b) Cronbach’s alpha between test items during each test round c) samples between pre-test and test d) samples between test and post-test
Avarage total^{a} (Cronbach Alpha$^{b)}$	16.5 (0.889)	42.4 (0.786)	39.4 (0.884)	
The significance of sign test for two related		$p<.001^{c}$	$p=.383^{d}$	

The results suggest that students learned the concept of a linear function, if “learning” is defined as in previous studies ([78], [79], [80], [81], [82], [83]). The positive results somewhat contradict the results of [84] that minimally guided instruction is less effective and less efficient than other instructional approaches (cf. [85]). In this light, we wanted to extend the methodology to find out what had actually happened during the learning processes, and what might have contributed to the positive results.

What influences did SCLE have on the students' mathematical identity?
The analysis of the Identity- and Maths-profiles (Q3) during ClassPad 2 project revealed huge differences in the profile shifts between the students [66]. The outcome suggests that the problem-solving processes during the project influenced students' Identity- and Maths-profiles. There is also a slight extending shift in the average profiles of the whole class. Eronen [1] (see p. 45 Figures 15 and 16) illustrates an example how Identity-profiles and Maths-profiles (see Section 1.2) change when students work in pairs. In that example, a conceptually-oriented peer-teacher (i.e., wanted to know what steps she has to undertake) was teaching her procedurally-oriented classmate (i.e. uninterested in understanding what he was doing; see [86]. This kind of peer-teaching period quasi-enriched the Maths-profile of the peer-teacher but undermined her mathematical self-confidence. Interestingly, the Identity-profiles of these two students seem to run in opposite directions. As the classmate begins to think (perhaps wrongly) that he can find, apply, and argue better than at the outset, his peer-teacher's own self-confidence in making mathematics seems to deteriorate. As the peer-teacher spent all her time in explanatory mode, this finding may indicate that a behaviourist approach to teaching can damage both student and teacher. In contrast, the teacher should be able to scaffold the learning process in the before-mentioned guidelines of SCL. This case might act as a warning against those who exaggerate the dominant role of emotional support and atmosphere (cf. [29]) because those two students were the best friends in the class.

How did the students communicate in SCLE?
The Data for Q4 were gathered from the eight student discussions during the project lessons. Discussions were audio recorded and data were transcribed. The audio data were analysed by classifying the major content of students' speech in each recorded minute. By focusing on interplay between formal and informal school culture (cf. [31], [33]) five categories of speech were identified: formal discussion about the problems, formal discussion about the technology used, silence, informal conversation concerning other school subjects and general informal discussion (entertainment) (see [1]). Table 2 shows the breakdown of conversation between one pair of students, 'John' and 'Sarah', over the course of the nine lessons. The average amount of informal and formal discussion was the same, but there was significant variation in time spent in each category during course of the project.

Table 2. Communication of two students, categorized by speech [77].

Categories (min)	**Informal entertainment**		**Silence**	**Formal discussion**		Total
	General subject	Regarding School		Regarding technology	Regarding the problems	
John	79	66	66	32	101	344
Sarah	77	58	46	26	137	344
Total	**41 %**		**16 %**	**43 %**		**100 %**

Figure 5 represents communication between two students (John and Sarah) during one lesson, being quite typical not only for these students but among the majority of the student pairs. The analysis revealed that the dominant category of communication was *entertainment.* This lesson was quite typical for students throughout the project. Both students went for Production tasks 1 to 4 during the first 19 minutes. This example demonstrates the significance of informal school when students work within SCLE, as well as the importance of storytelling and the potential of informal school culture activities as part of the learning process (cf. [33]).

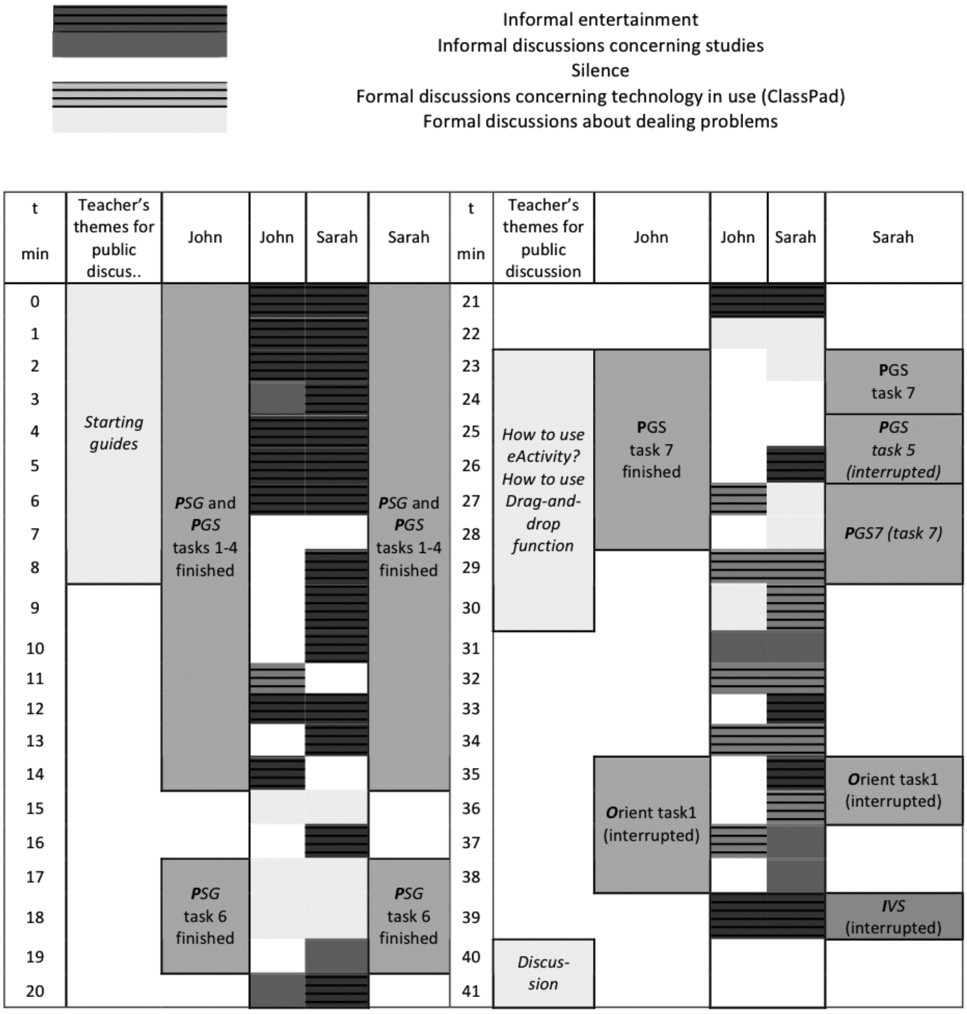

Figure 5. A sample of communication flow among a student pair. [1]

The communication between Sarah and John during the first six minutes concerned entertainment or their daily school activities. After that, both were silent for about one minute until Sarah started to communicate for three minutes for entertainment purposes. Figure 5 shows that the amount of discussion then decreased quite radically. We formed the view that this was caused by the difficulty of Production tasks that were not appropriate for this low-performing pair. The fact that after 23 minutes John stayed with Production Task #7 whereas Sarah attempted tasks #5 and #7 also indicates poor collaboration. After that, the teacher took more than 10 minutes to explain how to utilize ClassPad's eActivity, and the Drag-and-Drop properties of ClassPad. Students then selected Orientation Task #1 from the menu, and after that Identification Task IVS. However, the discussion kept returning to technological aspects or formal entertainment rather than mathematical content. The recording of this lesson (41 minutes) includes discussion of content (6 minutes), ClassPad (6 minutes), and studies in general (6 minutes), with 3 minutes of silence. The dominant category of speech (21 minutes, more than 50% of the total) was entertainment.

How did the students experience SCLE?

Data for Q5 were analysed by using the Grounded Theory method [76]. Students (N=23) wrote essays on their experiences, to be analysed using the "bottom-up" methods of grounded theory as described by Glaser and Strauss [87] and Glaser [88] (which introduced a more flexible approach). Quite rich data were obtained from the students' personal essays or reports, and three analytic steps were then followed. In the open coding phase, the data derived from the students' writings were examined in detail and coded for emerging concepts. During this open coding, concepts were identified. In the theoretical coding phase, the number of concepts was reduced and grouped into tentative categories. Patterns of students' processes were identified in the data, and the emerging categories and their relationships were carefully compared to ensure that these categories covered most of the variation found in the data. We constructed a model describing the process of mastering doing and learning mathematics through acquiring expertise processes. The processes were iterative, and mastery of doing and/or learning was reached either with satisfaction or dissatisfaction. Two different learning profiles, one concluding with students feeling satisfied with their learning and the other concluding with students feeling unsatisfied illustrated the students' typical processes. According to the model, the key elements for SCL are easy-to-use tools, shared understanding of the work style, and diverse tasks. In addition, students can set different goals for their mastery. Some students are satisfied with reaching the level of doing math, which might be a dissatisfying situation for a student who wants to learn mathematics (cf. [89]).

4. Grounding SCL through the ClassPad project

4.1 SCLE characterized by student's experiences

Recalling the SCLE, we begin with the fundamental questions 'Where do the learning tasks come from; who provides the motivation for doing them; which learning tools will be utilized; and who takes the control in the process?' Table 3 extends the discussion in Eronen & Kärnä [76] who describe the processes through which students acquire the expertise needed for mastering doing and learning mathematics

Table 3. Categories and sub-categories in doing tasks (D) and learning mathematics (L) during mathematics lessons.

		Mode of using the tool (T)	**Mode of working style (W)**	**Content-orientation (C)**
Self-confidence	**D**	Using the tool within instrumentation	Working properly	Doing tasks on the content
	L	Using the tool within Instrumentalisation	Learning orientation	Motivation to learn the content
Self-guidance	**D**	Self-guided using of the tool	Self-guided working	Self-guided task orientation
	L	Self-guided learning with the tool	Self- guided learning	Self-guided learning orientation
Minimal Instruction (MI)	**D**	Using the tool within MI	Working within MI	Task orientation
	L	Learning with the tool within MI	Working within MI aiming to learn	Learning the content within MI
Level of expertise	**D**	Shifting to instrumentalisation	Expert-like working	Expert-like investigation
	L	Utilizing instrumentalisation for	Expert-like learning strategies	Expert-like problem solving
Level of satisfaction in management processes (positive P vs. negative)	**D**	P/N	P/N	P/N
	L	P/N	P/N	P/N

The term "managing the learning" in a narrow behaviourist sense (cf. [70]) would mean that the teacher takes a full control of the learning process, giving out the tasks, showing how to solve them and use the tool, whilst applying our terminology from Section 1.4. In an ideal constructivist SCLE, it would mean that the student is able to cope within MI in the current investigation space allowing him or her to construct links between conceptual and procedural knowledge via instrumentalisation, keeping in mind the ability to apply this knowledge. Along this whole scale there are numerous variations but regarding SCLE emphasized in this article, we henceforth speak simply about ‘management of doing’ (MD) and ‘management of learning (ML)’, conducted by the student himself or herself.

The learning processes during the lessons was a complicated process shaped by a combination of factors illustrated in Figure 6, being developed from the model of Eronen & Kärnä [76]. Applying the notations from Table 3, the causal factor for MD and ML is the students’ self-confidence to learn mathematics. The conditional factor is the students’ ability to work and learn self-guided and collaboratively, whilst the teacher’s Minimalist Instruction strategy can be interpreted as a covariance factor. The core element of the process is derivable from the question ‘Can the student use the tool at the level of instrumentalisation within the Z-activities when handling the current investigation space? This means expert-like working and thinking and hence refines the term “expert acquisition” used as a core element of MD and ML. Regarding technology-based environments, it could be replaced by the term ”sustainable heuristics and instrumentalisation”, for example. It must be understood in a constructivist sense as being far from an acquisition of objectivist knowledge from outside the learner. All four modes of the management outcome in Figure 9 also included the satisfaction–dissatisfaction dimension, i.e. how satisfied or dissatisfied the student felt with his or her management regarding the tool, working style, and content subcategories.

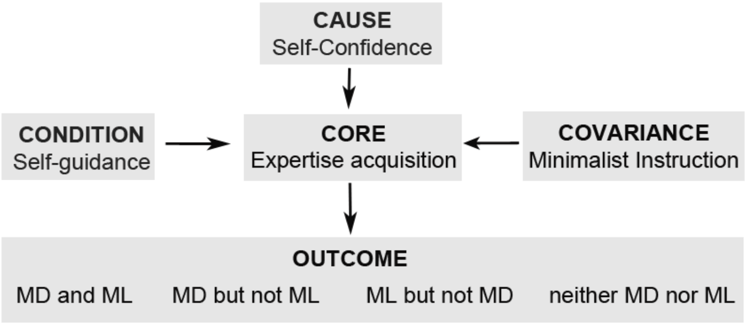

Figure 6. Development of the management of doing (MD) and learning (ML) in SCLE regarding the sub-categories of Table 3.

To give two examples, managing neither doing nor learning appeared typically in situations when the students stopped working. These situations became possible when the students had full autonomy to guide their own doing and learning. If the reason for entering this mode was due to personal or non-content-related issues shared among the classmates, the satisfaction increased, whilst decreased if the problem-solving process was unsuccessful. An opposite example comes from a student who had mathematical knowledge of the subject at the beginning of the learning period. This caused the outcome of ML without MD within a satisfaction mode. However, this kind of satisfaction decreased quickly as there was nothing to do or learn. Figure 7 illustrates two commonly found management processes in more detail by using V- and L-profiles.

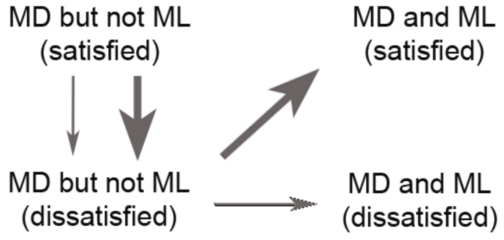

Figure 7. V-profile (bold arrows) and L-profile (thin arrows) for the learning management in SCLE (cf. [76]).

The V profile represents the process of LM for most of the students (n=21). The expert acquisition of these students was successful, and they were satisfied with their performance in all three subcategories (tool, work style, and content). At the beginning (from stage 1 to stage 2), the students were excited about this way of learning mathematics, which was very different from regular lessons, as the students were asked to use a new tool, and to complete self-guided work. After the first few lessons, however, the satisfaction level of V-profile students collapsed, as they faced difficulties in managing the tool and work style that slowed down the process of solving the assigned mathematics problems. This satisfaction collapse is traced in the transition from stage 2 to stage 3. As the project continued, these students acquired greater expertise in the tool and subsequently in the work style. The expert acquisition in the mathematical content (i.e. assigned mathematics problems) proved to be more difficult, usually taking several lessons. At the stage when the V-profile students understood for the first time how they could self-guide to solve problems by using the tool, this expertise accelerated with increasing awareness of how to use the new tool and work style for learning mathematics, and they moved from stage 3 to stage 4. The final outcome of the process for the V-profile students was that they managed their own doing and learning and felt satisfied.

The learning process of two other students is described by the L-profile. These students managed their learning but were dissatisfied at the end of the project. At the outset, both were as excited as the V-profile students. However, as the project progressed, the L-profile students had difficulties in expert acquisition regarding at least one of the components tool, working style, and content orientation. Two of the students who liked to work alone succeeded with T and W but not in C. The first one reported that some of the tasks were easy to solve using the tool, whilst some were too difficult to solve alone. The other one reported that although she completed all the tasks, she learned almost nothing during the project because of the new work style. However, she also mentioned that she had learned the concept of slope, which was crucial in understanding the equation of a straight line. So, the outcome for these L-profile students was that they managed their learning while being dissatisfied. However, at the same time they were dissatisfied if their learning process because it was guided by unilateral cooperation with peers or the teacher (see (3) in Figure 10). It became evident that for these two students, completion of stage 4 processes would require effective collaborative problem solving.

4.2. Conclusions

We now summarize the main outcomes regarding the research question Q of this article. Firstly, the findings suggest that optimal ‘student-centred learning’ emphasizes students’ freedom to choose learning objectives and working methods in problem-based socio-constructivist technology-based environments, in which open questions about both mathematics and technology are solved in collaboration between students or student teams. Even though the students proceeded more or less chaotically, it was found that learning to link conceptual and procedural knowledge might be organized successfully within a quasi-systematic framework. Although the students were worried at the outset about not having a teacher controlling their learning processes, the majority of the students

(N = 21) realized by the end that self-guidance facilitated rather than inhibited their doing and learning mathematics (Figure 9). As Eronen & Kärnä [76] emphasize, the critical elements of the SCL must be captured and understood in order to take the interrelationship between affect and cognition into account and to maximize the benefits of learning situations for all learners. According to our findings, a successful learning process includes different modes, and it shifts between these modes. During the process, the learner faces satisfying and dissatisfying moments, which are crucial for gaining a thorough understanding of the task to be learned. The students felt their feelings of satisfaction were stronger than their feelings of dissatisfaction, particularly after the students had been dissatisfied during the problem-solving process, and had a positive impact on the students' overall interest in doing and learning mathematics.

Secondly, our study suggests that the factors in the position of Swan [27] represented in Section 1.1 may even cause a failure to achieve positive shifts in instructional paradigms, as referred to by Lea et al. [90] in relation to the differences between educators' acts and thoughts. It might be appropriate to warn against over-romanticism regarding task-based classroom orchestration whereby the students are expected to learn mathematics just by finding answers to the tasks that the teacher has posed on specific and limited mathematical content in a well-defined closed form (cf. [50]). Dörner's well-known problem typology [28] makes it possible to come up with *dialectic problems* that disarm the negative emotions caused by the demand for objectivity. A teacher who has experience in utilizing this approach notices that the students usually convert these open problems to *interpolation problems* and even complicated *analysis-synthesis problems*. This happened in our ClassPad project when the students were asked, for example, to find out *how they think* a technical tool could help to create a representation on the algebra window from geometrical representations, and vice versa. This did not only vitalize the Z-activities in Figure 1 but opened up the complexity of the *on*, *with*, *in* and *between* paradigms, allowing for movement into the first row of Table 3. Student teams could choose — and, indeed, create — any object they wished. In this way, mathematical problems were seen as becoming important when they were psychologically meaningful for the students, as Haapasalo and Samuels [51] discuss in relation to educational robotics. This may be one reason behind the positive development in students' Maths-profiles and Identity-profiles. The study shows that autonomy and the opportunity to collaborate improves emotional climate, and also explains students' improved cognitive development (cf. [30], [15], [16], [17], [18]). The influence of informal school culture therefore becomes apparent in SCLE.

Thirdly, and particularly with the socio-constructivist paradigm in view (see Section 2.3), we now identify the SCLE by considering the following two crucial questions (cf. [91], [92]): (1) Who are the real actors in the learning process and what is the degree of collaboration? (2) Who are the primary decision makers in this process? We considered these questions on four levels to cover the core of the SCL process (see Table 4). Modifying and extending the terminology of Neuman [2], we mean by the paradigm *on* that the teacher is the main decision maker and person in charge, whilst the paradigm *with* allows students to participate more or less in the discussion of learning goals, materials, tools and working methods. The paradigm *in* means those determinants are assigned to individual students or to happen in student teams. In ideal socio-constructivist collaboration, the learning happens in negotiations between student teams and therefore the power regarding those determinants shifts from the individual students to the student teams. We therefore extend Neuman's [2] triple-step contexts by the paradigm *between* student teams. Table 4 illustrates one potential interpretation of this our categorization. A hypothetical example of interpreting the table is a student-oriented non-cooperative learning process whereby the teacher makes the decision *on* the learning goals and materials, whilst makes the decisions about learning tools and the process management at least to some extent together with the students. As this cell is not highlighted, we do not consider it to be SCLE.

Table 4. Matrix of the requirements for the crucial determinants of the student-centered learning environments (SCLE) categorized by four main characteristics: (G) goal setting and commitment, (L) learning material, (T) learning tools and (M) responsibility of process management.

	NON-COOPERATI	COOPERATIVE	COLLABORATION	COLLABORATION
STUDENT-CENTERED	**G: with/in** **L: with/in** **T: with/in** **M: in**	**G: with/in** **L: with/in** **T: in** **M: in**	**G: in** **L: in** **T: in** **M: in**	**G: between** **L: between** **T: between** **M: between**
STUDENT-ORIENTED	G: on L: on T: with M: with	G: on/with L: on/with T: with/in M: in	G: in L: with/in T: with/in M: in	G: in/between L: in/between T: in/between M: in/between
TEACHER-CENTERED	G: on L: on T: on M: on	G: on L: on T: on/with M: on/with	G: with/in L: with/in T: with/in M: with/in	G: with/between L: with/between T: with/in M: with/in

We feel the implications of this reappraisal to the whole school culture should be seriously considered. As the role of informal learning increases, not least because of progressive instrumental genesis, the learning focus could usefully be shifted from the classroom to students' free time activities as Haapasalo and Zimmermann [93] suggest:

> *which can stimulate modelling processes, for which school could take, referring to a car race, the role of a pit stop (to orchestrate technology-based investigation spaces which allow students to explore spontaneously the facility of real and virtual environments which are both, meaningful to them and their community, and which naturally motivate a greater use of mathematical language in its different forms.*

Our ongoing efforts to implement this pit stop culture in school and teacher education seem to be promising but difficult. With regard to the models developed through grounded theory, our further iterations of the ClassPad project focus on testing those models deductively in varied working environments, as well in schools and in teacher education.

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